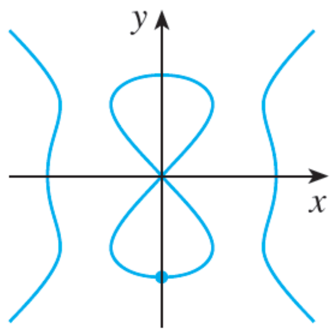
Exercise 32

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$y^2(y^2 - 4) = x^2(x^2 - 5), \quad (0, -2), \quad \text{(devil's curve)}$$



Solution

The aim is to evaluate y' at $x = 0$ and $y = -2$ in order to find the slope there. Differentiate both sides of the given equation with respect to x.

$$\frac{d}{dx}[y^2(y^2 - 4)] = \frac{d}{dx}[x^2(x^2 - 5)]$$

$$\left[\frac{d}{dx}(y^2)\right](y^2 - 4) + y^2\left[\frac{d}{dx}(y^2 - 4)\right] = \left[\frac{d}{dx}(x^2)\right](x^2 - 5) + x^2\left[\frac{d}{dx}(x^2 - 5)\right]$$

$$\left[(2y) \cdot \frac{d}{dx}(y)\right](y^2 - 4) + y^2\left[(2y) \cdot \frac{d}{dx}(y)\right] = (2x)(x^2 - 5) + x^2(2x)$$

$$(2y) \cdot (y')(y^2 - 4) + y^2(2y) \cdot (y') = 4x^3 - 10x$$

$$[2y(y^2 - 4) + 2y^3]y' = 4x^3 - 10x$$

Solve for y'.

$$y' = \frac{4x^3 - 10x}{2y(y^2 - 4) + 2y^3}$$

Evaluate y' at $x = 0$ and $y = -2$.

$$y'(0, -2) = \frac{4(0)^3 - 10(0)}{2(-2)[(-2)^2 - 4] + 2(-2)^3} = 0$$

Therefore, the equation of the tangent line to the curve represented by $y^2(y^2 - 4) = x^2(x^2 - 5)$ at $(0, -2)$ is

$$y + 2 = 0(x - 0).$$

Below is a graph of the curve and the tangent line at $(0, -2)$.